

Exercise 51

Let

$$B(t) = \begin{cases} 4 - \frac{1}{2}t & \text{if } t < 2 \\ \sqrt{t+c} & \text{if } t \geq 2 \end{cases}$$

Find the value of c so that $\lim_{t \rightarrow 2} B(t)$ exists.

Solution

In order for the limit to exist, the left-hand and right-hand limits must exist. That is, at $t = 2$ the two given functions must be equal.

$$\left(4 - \frac{1}{2}t\right)\Big|_{t=2} = \sqrt{t+c}\Big|_{t=2}$$

$$4 - \frac{1}{2}(2) = \sqrt{(2) + c}$$

Solve for c .

$$3 = \sqrt{2+c}$$

Square both sides.

$$9 = 2 + c$$

Therefore,

$$c = 7.$$